MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2022/2023

PEM0026 – TRIGONOMETRY & GEOMETRY

(Foundation in Engineering)

13 JULY 2023
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **FOUR (4) pages** excluding cover page with **FOUR (4) questions** only.
2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the Answer Booklet provided. All necessary working **MUST** be shown.
4. Only **non-programmable** calculator is allowed.

QUESTION 1 [25 marks]

(a) Find the exact value of the following expressions. Do not use calculator.

(i) $\dfrac{\cos 10^\circ}{\sin 80^\circ}$ (1.5 marks)

(ii) $\sec 23^\circ - \csc 67^\circ$ (1.5 marks)

(b) If $\sin\theta = \dfrac{2}{5}$ and θ is an acute angle, find the value of $\cos\theta$ using the following methods:

(i) Right triangle diagram (1.5 marks)

(ii) Trigonometric identity (1.5 marks)

(c) Graph $y = 3\cos(0.5x)$ using the following methods:

(Use radian value in all graphs and computations)

(i) Key-points. Sketch the graph for $0 \le x \le 4\pi$. (6.5 marks)

(ii) Transformations. Include points $(0,1), (\pi,-1), (2\pi,1), (3\pi,-1), (4\pi,1)$ for the first stage of the graph ($y = \cos(x)$). (6.5 marks)

(d) Solve the triangle in **Figure 1** below: (6 marks)

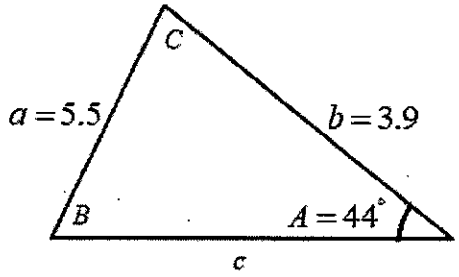

Figure 1

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QUESTION 2 [25 marks]

(a) Find the exact value of the following composite functions:

(i) $\sin^{-1}\left(\sin\frac{4}{7}\pi\right)$ (3 marks)

(ii) $\tan\left(\cos^{-1}\left(-\frac{2}{3}\right)\right)$ (3 marks)

(b) Establish the identity: $\frac{\cos^2\theta-\cot\theta}{\sin^2\theta-\tan\theta}=\cot^2\theta$ (9 marks)

(c) Solve the equation $-\tan(2\theta)+2\sin\theta=0$ on the interval $0\le\theta<2\pi$. (10 marks)

QUESTION 3 [25 marks]

(a) If $w_1=\frac{1}{2}+i\frac{2}{5}$ and $w_2=0.25+i4.50$, determine the following:

(i) w_1w_2 in polar form (3.5 marks)

(ii) $\frac{w_1}{w_2}$ in polar form (3.5 marks)

(b) If $z_1=3\cos\left(\frac{\pi}{12}\right)+i3\sin\left(\frac{\pi}{12}\right)$ and $z_2=4\cos\left(\frac{\pi}{8}\right)+i4\sin\left(\frac{\pi}{8}\right)$, determine the following:

(i) z_1z_2 in rectangular form (2 marks)

(ii) $\frac{z_1}{z_2}$ in rectangular form (2 marks)

(iii) $\left(\frac{z_1}{z_2}\right)^5$ in rectangular form (2 marks)

(c) Given $\mathbf{a}=2i+3j-5k$, $\mathbf{b}=6i-4j+k$ and $\mathbf{c}=4i+6j-10k$. Do the following:

(i) Calculate the distance from **a** to **b**. (1 mark)

(ii) Find the unit vector in the direction of **c**. (2 marks)

(iii) Determine whether **c** is parallel to **a**. (2 marks)

(iv) Check whether **a** and **b** are orthogonal, parallel or neither. Next, find the angle between the two vectors. (4 marks)

(v) Compute a vector that is orthogonal to both **a** and **b**. (3 marks)

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QUESTION 4 [25 marks]

(a) An equation of a parabola is given as $y = \frac{4}{3}x^2$. Do the following:

(i) Find the focus, directrix and the length of the latus rectum of the parabola. (6 marks)

(ii) Sketch the parabola. Label the focus, directrix and latus rectum. (5 marks)

(b) A hyperbola is shown in **Figure 2**.

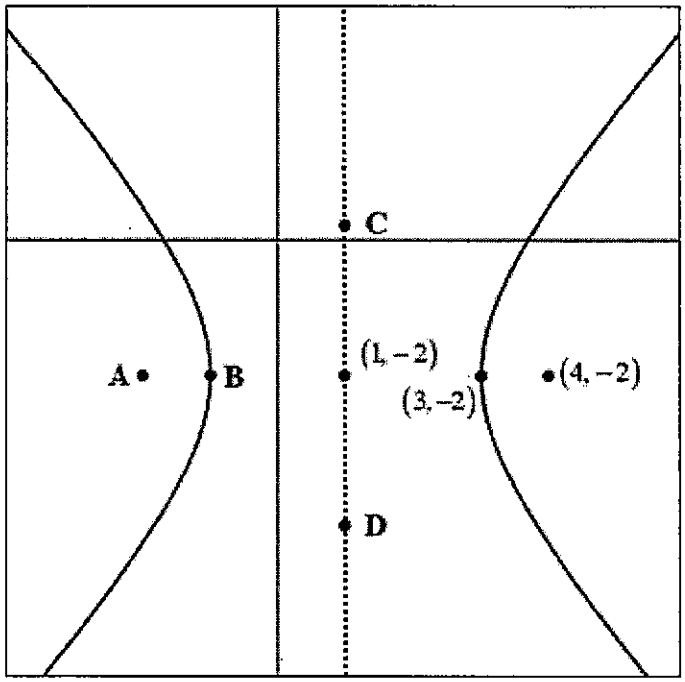

Figure 2

The center of the hyperbola is at $(1,-2)$, one focus at $(4,-2)$ and one vertex at $(3,-2)$. Determine the equation of the hyperbola. Next, find the coordinate for point **A**, **B**, **C** & **D**. (14 marks)

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APPENDIX; TRIGONOMETRY IDENTITIES

$\sin^2\theta+\cos^2\theta=1$; $\sec^2\theta=1+\tan^2\theta$; $\csc^2\theta=1+\cot^2\theta$

$\cos 2\theta=\cos^2\theta-\sin^2\theta$; $\cos 2\theta=2\cos^2\theta-1$; $\cos 2\theta=1-2\sin^2\theta$

$\sin 2\theta=2\sin\theta\cos\theta$; $\tan 2\theta=\dfrac{2\tan\theta}{1-\tan^2\theta}$

$\sin(x\pm y)=\sin x\cos y\pm\cos x\sin y$ $\cos(x\pm y)=\cos x\cos y\mp\sin x\sin y$

$$\tan(x\pm y)=\frac{\tan x\pm\tan y}{1\mp\tan x\tan y}$$

$2\sin A\cos B=\sin(A+B)+\sin(A-B)$

$2\cos A\cos B=\cos(A+B)+\cos(A-B)$

$2\sin A\sin B=\cos(A-B)-\cos(A+B)$

$\sin A+\sin B=2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$; $\sin A-\sin B=2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$

$\cos A+\cos B=2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$; $\cos A-\cos B=-2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$

$\sin^2\dfrac{A}{2}=\dfrac{1-\cos A}{2}$; $\cos^2\dfrac{A}{2}=\dfrac{1+\cos A}{2}$; $\tan^2\dfrac{A}{2}=\dfrac{1-\cos A}{1+\cos A}$

$\sin\dfrac{A}{2}=\pm\sqrt{\dfrac{1-\cos A}{2}}$; $\cos\dfrac{A}{2}=\pm\sqrt{\dfrac{1+\cos A}{2}}$; $\tan\dfrac{A}{2}=\pm\sqrt{\dfrac{1-\cos A}{1+\cos A}}=\dfrac{1-\cos A}{\sin A}=\dfrac{\sin A}{1+\cos A}$

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